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THERMOELASTICITY OF NONHOMOGENEOUS MEDIA

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A system of equations is derived for the coupled thermoelasticity of an anisotropic nonhomogeneous body, taking into account the generalized law of heat conduction, by the method of systems identification and with the aid of the Clausius–Duhem inequality.

In view of the extensive use of composite materials in various branches of technology, it becomes very important to study the properties of nonhomogeneous media.

The process of heat propagation through a nonhomogeneous medium will be simulated as follows: A system with the transfer function $G_{ij}(x_s)$, representing an anisotropic nonhomogeneous medium, receives a temperature gradient ∇t at the input and transmits a thermal flux $\vec{q}$ as its output signal. The output $\vec{q}$ of a linear process with the input ∇t is determined as the convolution integral

$$q_i = \int_0^{\tau} \nabla t(x_s,\ \tau - \tau_1)\, G_{ij}(x_s,\ \tau_1)\, d\tau_1 \tag{1}$$

or

$$q_i = \int_0^{\tau} \nabla t(x_s,\ \tau_1)\, G_{ij}(x_s,\ \tau - \tau_1)\, d\tau_1. \tag{2}$$

We introduce the notation $Z = q_i$ and $Y = \nabla t$. We then define the correlation function φ_{zy} which describes the coupling between quantities Z and Y

$$\varphi_{zy} = \lim_{\tau_3 \to \infty} \frac{1}{2\tau_3} \int_{-\tau_3}^{\tau_3} Z(x_s,\ \tau)\, Y(x_s,\ \tau - \tau_1)\, d\tau. \tag{3}$$

We analogously define the autocorrelation function φ_{yy} as the average product of the value of signal $Y(x_s,\ \tau)$ and its value at time $(\tau - \tau_2)$

$$\varphi_{yy} = \lim_{\tau_3 \to \infty} \frac{1}{2\tau_3} \int_{-\tau_3}^{\tau_3} Y(x_s,\ \tau)\, Y(x_s,\ \tau - \tau_2)\, d\tau. \tag{4}$$

We will consider an input function Y of the "white noise" kind, whose autocorrelation function is a delta function. A preliminary transformation of function (3) with relation (1) taken into account yields

$$\varphi_{zy}(x_s,\ \tau_2) = G_{ij}(x_s,\ \tau_2). \tag{5}$$

With the aid of relation (5), relation (2) transforms to

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$$Z = \int_0^\tau Y(x_s, \tau_1)\,\varphi_{zy}(x_s, \tau - \tau_1)\,d\tau_1. \tag{6}$$

If the thermal flux and the temperature gradient are noncorrelated functions, then the correlation function will be defined as

$$\varphi_{zy}(x_s, \tau - \tau_1) = -\lambda_{ij}^t(x_s)\,\delta(\tau - \tau_1). \tag{7}$$

From relations (6) and (7) we obtain the Fourier law of heat conduction for a nonhomogeneous sphere

$$q_i = -\lambda_{ij}^t(x_s)\,\nabla t(x_s, \tau). \tag{8}$$

If the thermal flux and the temperature gradient are stationary random functions, then the correlation function φ_{zy} will be defined as

$$\varphi_{zy}(x_s, \tau - \tau_1) = \exp\left[-\frac{\tau - \tau_1}{\tau_r(x_s)}\right]\lambda_{ij}^t(x_s). \tag{9}$$

From expressions (6) and (9) we obtain

$$q_i + \tau_r(x_s)\frac{\partial q_i}{\partial \tau} = -\lambda_{ij}^t(x_s)\,\nabla t, \tag{10}$$

which is the generalized law of heat conduction characterizing the propagation of heat at a finite velocity through an anisotropic nonhomogeneous medium.

The derivation of these laws of heat conduction for a nonhomogeneous medium (their special cases being laws of heat conduction for a homogeneous medium) indicates the possibility of using methods in the identification theory for the analysis of heat conduction problems.

Mechanical or thermal actions during deformation of a body produce a coupling effect due to interaction of the strain field and the temperature field. This effect manifests itself in the appearance of thermoelastic waves and thermoelastic dissipation of energy. We will derive the equations of linear coupled thermoelasticity on the basis of two thermodynamic postulates — the law of energy conservation and the Clausius–Duhem inequality. The physicomechanical characteristics of nonhomogeneous media are continuous functions of the coordinates.

The local condition of energy conservation is expressed by the equation

$$(\dot f + \dot\Theta\cdot S + \Theta\cdot\dot S) - \sigma_{ij}\dot e_{ij} + q_{i,i} + W_t = 0. \tag{11}$$

The local Clausius–Duhem inequality is

$$-S\dot t - \dot f + \sigma_{ij}\dot e_{ij} - q_i(t_{,i}/t_0) \geqslant 0, \tag{12}$$

where $t = (\Theta - t_0)$ denotes temperature increments at points of the body.

Using the expansion of free energy $f(e_{ij}, \Theta)$ into a power series in e_{ij} and Θ in the vicinity of the natural state $f(0, t_0)$ [2], we express this free energy as

$$f(e_{ij}, \Theta) = \frac{1}{2}\,[C_{ijkl}(x_s)\,e_{ij}e_{kl} - 2\beta_{ij}(x_s)\,e_{ij}t + m(x_s)\,t^2]. \tag{13}$$

Inserting expression (13) into the inequality (12) yields

$$-[S - \beta_{ij}(x_s)\,e_{ij} + m(x_s)t]\cdot\dot t + [\sigma_{ij} - C_{ijkl}(x_s)\,e_{kl} + \beta_{ij}(x_s)\,t]\cdot\dot e_{ij} - C_{ijkl}(x_s)\,e_{ij}\,\dot e_{kl} - q_i(t_{,i}/t_0) \geqslant 0. \tag{14}$$

Inequality (14) must hold true for any values of $\dot e_{ij}$ and $\dot t$, which requires that the coefficients of $\dot e_{ij}$ and $\dot t$ in expression (14) become equal to zero. Then

$$\sigma_{ij} = C_{ijkl}(x_s)\,e_{kl} - \beta_{ij}(x_s)\,t, \tag{15}$$

$$S = \beta_{ij}(x_s)\,e_{ij} - m(x_s)\,t. \tag{16}$$

For an explanation of the physical meaning of $m(x_s)$ we will use the well-known relation of thermodynamics

$$\Theta\left(\frac{\partial S}{\partial \Theta}\right)_{e_{ij}} = C_e(x_s). \tag{17}$$

Differentiation of expression (16) with respect to Θ and multiplication of the result by Θ yields

$$m(x_s) = -C_e(x_s)\frac{1}{\Theta}. \quad (18)$$

Inserting expression (18) into expression (16) and expanding t/Θ into a series, of which we retain only the first term, yields

$$S = \beta_{ij}(x_s)e_{ij} + C_e(x_s)\left(\frac{t}{t_0}\right). \quad (19)$$

We now apply the operator $l = [1 + \tau_r(x_s)\partial/\partial\tau]$ to both sides of Eq. (11). As a result, the latter becomes

$$l[(\dot{f} + \Theta\dot{S} + \dot{\Theta}S) - \sigma_{ij}\dot{e}_{ij} + q_{i,i} + W_t] = 0. \quad (20)$$

Inserting expressions (13) and (19) into Eq. (20), taking also into account the generalized law of heat conduction (10), we obtain

$$l[\beta_{ij}(x_s)t_0\dot{e}_{ij} + C_e(x_s)\dot{t}] = [\lambda_{ij}^t(x_s)t_{,j}]_{,i} + lW_t - l[C_{ijkl}(x_s)e_{ij}\dot{e}_{kl}]. \quad (21)$$

Letting the dissipation term $C_{ijkl}(x_s)e_{ij}\dot{e}_{kl} = 0$, inasmuch as we are considering a thermoelastic medium, we obtain the generalized equation of heat conduction

$$l[\beta_{ij}(x_s)t_0\dot{e}_{ij} + C_e(x_s)\dot{t}] = [\lambda_{ij}^t(x_s)t_{,j}]_{,i} + lW_t. \quad (22)$$

The differential equation of motion is

$$\sigma_{ij,j} + X_i = \rho(x_s)\ddot{u}_i. \quad (23)$$

Inserting expression (15) into Eq. (23) and using the relation

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

we obtain

$$[C_{ijkl}(x_s)u_{k,l}]_{,j} - [\beta_{ij}(x_s)t]_{,j} + X_i = \rho(x_s)\ddot{u}_i. \quad (24)$$

The system of differential equations of coupled thermoelasticity for an anisotropic nonhomogeneous body is

$$[C_{ijkl}(x_s)u_{k,l}]_{,j} - [\beta_{ij}(x_s)t]_{,j} + X_i = \rho(x_s)\ddot{u}_i, \quad (25)$$

$$l[\beta_{ij}(x_s)t_0\dot{e}_{ij} + C_e(x_s)\dot{t}] = [\lambda_{ij}^t(x_s)t_{,j}]_{,i} + lW_t. \quad (26)$$

From this system we obtain the system of equations of uncoupled thermoelasticity for an anisotropic nonhomogeneous body

$$[C_{ijkl}(x_s)u_{k,l}]_{,j} - [\beta_{ij}(x_s)t]_{,j} + X_i = \rho(x_s)\ddot{u}_i, \quad (27)$$

$$l(C_e(x_s)\dot{t} = [\lambda_{ij}^t(x_s)t_{,j}]_{,i} + lW_t. \quad (28)$$

The preceding systems of equations of coupled thermoelasticity for anisotropic nonhomogeneous bodies can serve as a basis for analyzing the behavior of composite materials under various pulse loads.

NOTATION

x_s (s = 1, 2, 3), Cartesian rectangular coordinates; τ, time; λ_{ij}^t (i, j = 1, 2, 3), thermal conductivities of an anisotropic body; $\delta(\tau)$, Dirac function; τ_r, relaxation time for thermal flux; t_0, temperature of a body in the unstressed state; Θ, absolute temperature at points of a body; W_t, density of internal heat sources; S, entropy; σ_{ij}, components of the stress tensor in numbered Cartesian coordinates; e_{ij}, components of the strain tensor in numbered Cartesian coordinates; C_{ijkl}, elasticity coefficients for an anisotropic nonhomogeneous body (k, l = 1, 2, 3); β_{ij}, coefficients representing mechanical and thermal properties of the material of an anisotropic nonhomogeneous body; C_e, heat capacity per volume at a constant strain; u_i, components of the displacement vector in numbered Cartesian coordinates; X_i, components of the vector of body forces; and ρ, density of an anisotropic nonhomogeneous body.

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